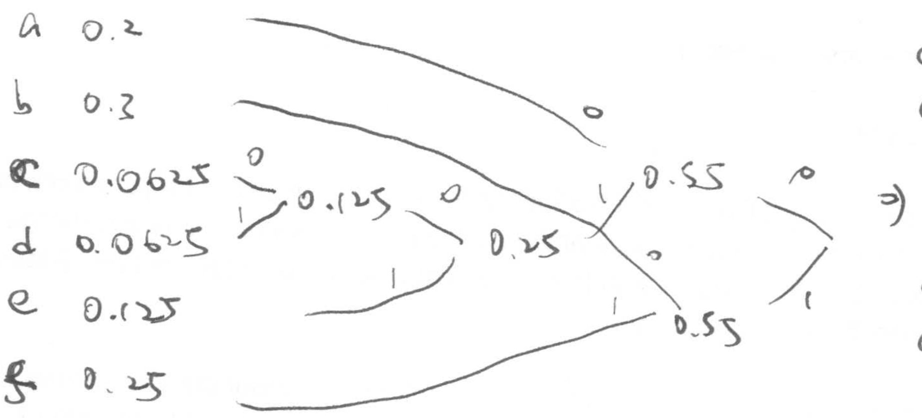
HW 3

1 a).



⇒

$C(a) = 00$
$C(b) = 10$
$C(c) = 0100$
$C(d) = 0101$
$C(e) = 011$
$C(f) = 11$

b). $E(L) = 0.2 \cdot 2 + 0.3 \cdot 2 + 0.0625 \cdot 4 + 0.0625 \cdot 4 + 0.125 \cdot 3 + 0.25 \cdot 2$
$= 2.375$ bits

c).

$p(x)$	$F(x)$	$\bar{F}(x)$		$l(x) = \lceil \log \frac{1}{p(x)} \rceil + 1$	$C(x)$
0.2	0.2	0.1	0.0001	4	0001
0.3	0.5	0.35	0.0101	3	010
0.0625	0.5625	0.53125	0.10001	5	10001
0.0625	0.625	0.59375	0.10011	5	10011
0.125	0.75	0.6875	0.1011	4	1011
0.25	1	0.875	0.111	3	111

d). $E(L) = 0.2 \cdot 4 + 0.3 \cdot 3 + 0.0625 \cdot 5 + 0.0625 \cdot 5 + 0.125 \cdot 4 + 0.25 \cdot 3$
$= 3.575$ bits

e). $H(X) = -\sum p(x) \log p(x) = 2.36$ bits

2). $H = \left[\begin{array}{ccc|cccc} 1 & & & 1 & 1 & & 1 \\ & 1 & & 1 & & 1 & 1 \\ & & 1 & & 1 & 1 & 1 \end{array} \right]$ with $\underbrace{}_{P}$ under the right block, $G = \begin{pmatrix} P \\ I \end{pmatrix}$

a). encoded message $= G \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

b). $[1001011]^t$

c). $S = H [1001011]^t = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

The decoder will think the 4th bit is flipped.
Therefore, it will think $[1000011]^t$ was sent

d). The message is the last 4 bits, i.e. $[0011]^t$